AP Calculus AB
2000 Student Samples

CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

R_1

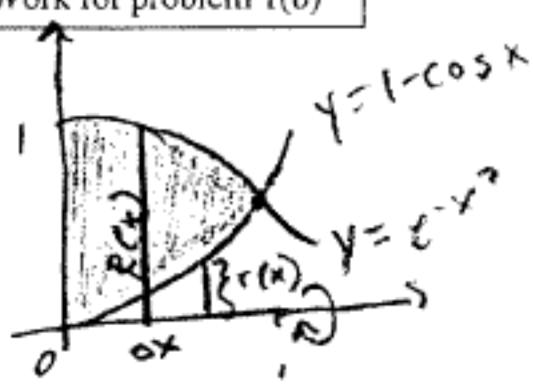
A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$$A = \int_0^{.94194408} (e^{-x^2}) - (1 - \cos x)\, dx$$

$$A \approx \boxed{.5907 \text{ units}^2}$$

Work for problem 1(b)



$$V = \pi \int_0^{.94194408} R^2(x) - r^2(x)\, dx$$

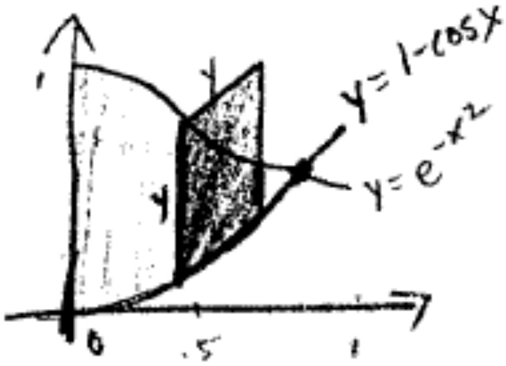
$$V = \pi \int_0^{.94194408} (e^{-x^2})^2 - (1 - \cos x)^2\, dx$$

$$\boxed{V \approx 1.7466 \text{ units}^3}$$

Continue problem 1 on page 5.

R_2

Work for problem 1(c)



$$V = \int \left[e^{-x^2} - (1-\cos x)\right]^2 dx$$

$y = e^{-x^2} - (1-\cos x)$

$y^2 = \left[e^{-x^2} - (1-\cos x)\right]^2$

$$\boxed{V \approx .4611 \text{ units}^3}$$

T1

CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

To find the right hand limit of integration I graphed the two functions and then used the intersect function on my calculator.

$$A = \int_0^{.9419} e^{-x^2} - (1-\cos x) = \boxed{.591}$$

Work for problem 1(b)

$$V = \pi \int_0^{.9419} (e^{-x^2})^2 - (1-\cos x)^2 = \boxed{.556\pi \text{ or } 1.75}$$

Continue problem 1 on page 5.

T_2

Work for problem 1(c)

$$v = \int_0^{.9419} \left(e^{-x^2} - (1-\cos x)\right)^2 = \boxed{.4223}$$

$A = s^2$

$s = e^{-x^2} - (1-\cos x)$

GO ON TO THE NEXT PAGE.

CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

W_1

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$$A = \int_0^1 e^{-x^2} - (1-\cos x)\, dx$$

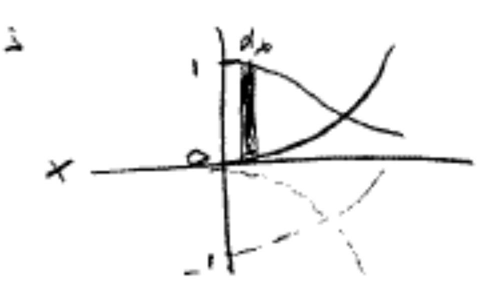
$$A = 0.58829\ u^2$$

Work for problem 1(b)

revolution about x-axis

$$V = \pi \int_0^1 \left[e^{-x^2} - (1-\cos x)\right]^2 dx$$

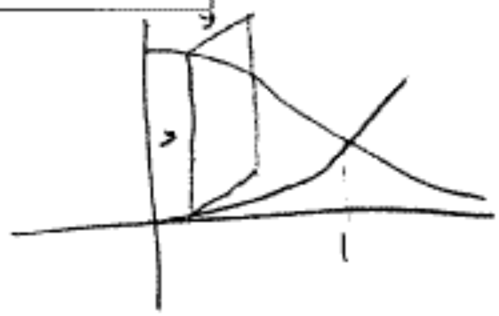
$$V = 1.44899\ u^3$$



Continue problem 1 on page 5.

W_2

Work for problem 1(c)



$$V = \int_0^1 \left[e^{-x^2} - (1-\cos x)\right]^2 dx$$

$V = 0.46123 \ u^3$

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A₁

Work for problem 2(a)

Runner A

$m = \frac{10-0}{3-0} = \frac{10}{3}$

$v - 10 = \frac{10}{3}(t-3)$

$v(t) = \frac{10}{3}t$

$v(2) = \frac{10}{3}(2)$

$\boxed{= 6.667 \text{ m/s}}$

Runner B

$v(t) = \frac{24t}{2t+3}$

$v(2) = \frac{24(2)}{2(2)+3}$

$\boxed{= 6.857 \text{ m/s}}$

Work for problem 2(b)

Runner A

$v(t) = \frac{10}{3}t$

$a(t) = v'(t) = \frac{10}{3}$

$\boxed{a(2) = 3.333 \text{ m/s}^2}$

Runner B

$v(t) = \frac{24t}{2t+3}$

$a(t) = v'(t) = \frac{(2t+3)(24) - (2)(24t)}{(2t+3)^2}$

$= \frac{48t + 72 - 48t}{(2t+3)^2}$

$= \frac{72}{(2t+3)^2}$

$a(2) = \frac{72}{(2(2)+3)^2}$

$= \frac{72}{49} = \boxed{1.469 \text{ m/s}^2}$

Continue problem 2 on page 7.

A_2

Work for problem 2(c)

Runner A

Total Distance $= \int_0^{10} v(t)\,dt$

$= \int_0^3 \left(\frac{10}{3}t\right) dt + \int_3^{10} (10)\,dt$

$= 15_m + 70_m$

$\boxed{= 85\text{ m}}$

Runner B

Total Distance $= \int_0^{10} v(t)\,dt$

$= \int_0^{10} \left(\frac{24t}{2t+3}\right) dt$

$\boxed{= 83.336\text{ m}}$

C, 1

Work for problem 2(a)

velocity of runner b

a. Velocity of Runner A =
(0,0) (3,10)

$\frac{10-0}{3-10} = 10/3 = m$

(3,10) (2,y) =

$\frac{y-10}{2-3} = 10/3$

$\boxed{= 6.67\ m/s}$

b. $v(t) = \frac{24t}{2t+3}$

$v(2) = \frac{(24)(2)}{2(2)+3}$

$= \boxed{6.857\ m/s}$

Work for problem 2(b)

a Runner A = ?

$v/t = a$

$V_a = 6.67\ m/s$
$t = 2s$

$\boxed{\frac{6.67}{2} = 3.33\ m/s^2}$
acceleration of runner A

a of runner b = ?

$a = v/t$

$v = 6.857\ m/s$

$t = 2s$

$\frac{6.857}{2} = 3.4285$

$\boxed{3.4285\ m/s^2}$
acceleration of runner b

Continue problem 2 on page 7.

C_2

Work for problem 2(c)

Runner A

$d = vt$

area under curve

from $0, 10$

$= \frac{1}{2}(3 \cdot 10) + 7(10)$

$\boxed{85\text{ m}}$

Runner A

Runner B

$\int v(t) = d(t)$

$\int_0^{10} \frac{24t}{2t+3} =$

$\boxed{83.336\text{ m}}$

Runner B

GO ON TO THE NEXT PAGE.

F_1

Work for problem 2(a)

Runner A graph : v(t) →

Runner B $v(t) = \frac{24t}{2t+3}$

Runner A

from the graph $v(2) = 7$ m/s

Runner B

$v(2) = \frac{24(2)}{2(2)+3}$

$= \frac{48}{7}$

$= 6.85$ m/s

Work for problem 2(b)

Runner A

$(2,7)$ & $(3,10)$

acceleration at $t=2$ $= \frac{10-7}{3-2}$

$= 3$ m/s^2

Runner B

$v(t) = \frac{24t}{2t+3}$

$a(t) = \frac{(24)(2t+3)-(24t)(2)}{(2t+3)^2}$

$= \frac{48t+72-48t}{(2t+3)^2}$

$a(2) = \frac{72}{(2(2)+3)^2}$

$= \frac{72}{(7)^2}$

$= 1.47$ m/s^2

Continue problem 2 on page 7.

F_2

Work for problem 2(c)

Total distance covered by Runner A = area under the v-t graph

$= \frac{1}{2}(\cancel{10}^{5})(3) + (7)(10)$

$= 15 + 70$

$= 85\, m$

Total distance covered by Runer B $= \int_0^{10} v(t)\, dt$

$= \int_0^{10} \frac{24t}{2t+3}\, dt$

$= 83.3\, m$

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The College Board is a national nonprofit membership association dedicated to preparing, inspiring, and connecting students to college and opportunity. Founded in 1900, the association is composed of more than 3,900 schools, colleges, universities, and other educational organizations. Each year, the College Board serves over three million students and their parents, 22,000 high schools, and 3,500 colleges, through major programs and services in college admission, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT®, the PSAT/NMSQT™, the Advanced Placement Program® (AP®), and Pacesetter®. The College Board is committed to the principles of equity and excellence, and that commitment is embodied in all of its programs, services, activities, and concerns.

A, 1

Work for problem 3(a)

a. minimum - (-1,)

Candidate theorem says f may have a minimum on an open interval where $f'(x)=0$ or does not exist. $f'(x)$ exists everywhere and equals 0 at -5, -1, and 5. $f'(x)$ changes sign at -1 from negative to positive, which means $f(x)$ has stopped decreasing and started increasing. This is the only candidate to change sign in this manner.

Work for problem 3(b)

b. Maximum - (-5)

The Candidate theorem says that f may have a maximum on an open interval where $f'(x)=0$ or does not exist. $f'(x)$ exists everywhere and equals 0 at -5, -1, and 5. $f'(x)$ changes sign from negative to positive at -1, does not change sign at 5, and changes sign from positive to negative at -5. Sign change from positive to negative indicates a relative maximum, because $f(x)$ has stopped increasing and started decreasing.

Continue problem 3 on page 9.

Work for problem 3(c)

c. $f''(x) < 0$ - $-7 < x < -3$, $2 < x < 3$, $3 < x < 5$

Work for problem 3(d)

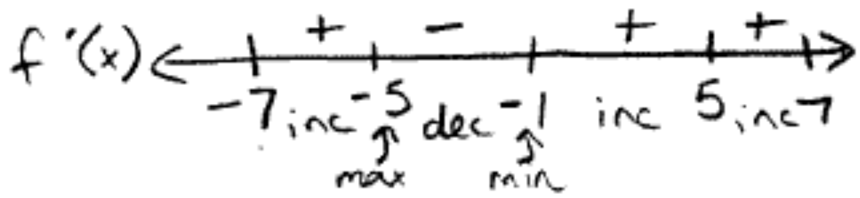
d. max - (7)

The Candidate Theorem tells us that when f may have its absolute maximum when $f'(x) = 0$ and at the endpoints of a closed interval. This leaves us (-7, -5, -1, 5, 7). f(x) must be increasing up to the max (eliminates -7 and -1) and cannot increase more immediately after the max (eliminates 5). We know f(x) is increasing when f'(x) is posi
To decide between -5 and 7, we can see that there is more increase of f(x) (positive area under the curve of f'(x)) than decrease (negative area) between -5 and 7
So f(x) must have increased more than decreased between -5 and 7, making f(7) larger than f(-5)

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Work for problem 3(a)



f attains relative minima at $x = -7$, because it is an end point from which f increases, and at $x = -1$, because f' changes from negative to positive here, indicating a change in f from decreasing to increasing, indicative of a local minimum

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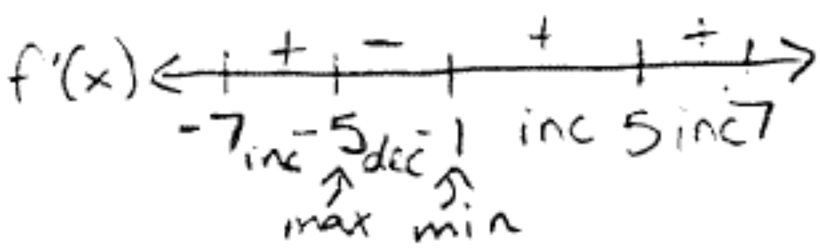
Work for problem 3(b)



f attains relative maxima at $x = 7$, because it is an end point of the function to which f increases, and at $x = -5$, because f' changes from positive to negative here, indicating a change in f from increasing to decreasing, indicative of a local maximum

Continue problem 3 on page 9.

Work for problem 3(c)

$f'(x)$ decreases $(-7,-3) \cup (2,5)$
$f'(x)$ increases $(-3,2) \cup (5,7)$

$f''(x)<0$ where $f'(x)$ decreases

$\therefore f''(x)<0$ on the intervals $(-7,-3) \cup (2,5)$

Work for problem 3(d)

$x = 7$

Absolute maxima occur where $f'(x)=0$ and is changing from positive to negative OR at the end points of the function. The area under the given curve $f'(x)$, which is $f(x)$, increases the most $(-1,7)$, more than $(-7,-5)$ where the other local maximum occurs at $x=-5$. Thus the absolute maximum occurs at $x=7$.

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Work for problem 3(a)

at $x=-1$, f attains a relative minimum, because $f'(-1)=0$ and $f'(x) \rightarrow \underset{-1}{- \mid +}$
therefore at $x=-1$ there is a relative minimum

Work for problem 3(b)

at $x=-5$, f attains a relative maximum because $f'(-5)=0$ and
$f'(x) \rightarrow \underset{-5}{+ \mid -}$, therefore at $x=-5$ there is a relative maximum

Continue problem 3 on page 9.

Work for problem 3(c)

$\left.\begin{matrix}[-7,-3] \\ [2,5]\end{matrix}\right\} f''(x)<0$

The graph of f(x) is concave down during these intervals

Work for problem 3(d)

at x=7 the graph of f attains its absolute maximum because ~~the~~ f'(x) has been positive for most of the interval, therefore at x=7, f attains its highest point.

END OF PART A OF SECTION II

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B₁

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

WATER LEAKS OUT AT A RATE OF:

~~[illegible]~~

$\frac{dv}{dt}$ } rate for leakage

$$\frac{dv}{dt} = -\sqrt{t+1} \text{ gallons/min.}$$

$$\int_0^3 dv = \int_0^3 -\sqrt{t+1}\,dt$$

$$V\Big]_0^3 = -\left[\frac{2(t+1)^{\frac{3}{2}}}{3}\right]_0^3 = -\left(\frac{2(8)}{3} - \frac{2}{3}\right)$$

$$= -\frac{14}{3} \text{ gallons}$$

$$\boxed{\frac{14}{3} \text{ gallons leak out}}$$

Work for problem 4(b)

rate at which volume in tank is changing $\rightarrow$ $\frac{dv}{dt} = 8 - \sqrt{t+1}$ gallons/min.

$$\int_0^3 dv = \int_0^3 (8 - \sqrt{t+1})\,dt$$

$$V\Big]_0^3 = \left[8t - \frac{2(t+1)^{\frac{3}{2}}}{3}\right]_0^3$$

$$= \left(24 - \frac{2(8)}{3}\right) - \left(-\frac{2}{3}\right)$$

$$= \frac{56}{3} + \frac{2}{3} = \boxed{\frac{58}{3} \text{ gallons}}$$

72
16
56

INITIAL

$$\frac{58}{3} + 30$$

$$\boxed{= \frac{148}{3} \text{ gallons}}$$

ANSWER

Continue problem 4 on page 11.

B_2

Work for problem 4(c)

$$\frac{d}{dt} A(t) = 8 - \sqrt{t+1}$$

$$\int d\, A(t) = \int \left(8 - \sqrt{t+1}\right) dt$$

$$A(t) = 8t - \frac{2(t+1)^{\frac{3}{2}}}{3} + C$$

$$30 = 8(0) - \frac{2(0+1)^{\frac{3}{2}}}{3} + C$$

$$C = 30 + \frac{2}{3} = \frac{92}{3}$$

$$\boxed{A(t) = 8t - \frac{2(t+1)^{\frac{3}{2}}}{3} + \frac{92}{3}}$$

Work for problem 4(d)

$$8 - \sqrt{t+1} = 0$$

$$-\sqrt{t+1} = -8$$

$$\sqrt{t+1} = 8$$

$$t+1 = 64$$

$$t = 63$$

$A'(t)$ + −

$A(t)$ INC 63 DEC t

AT $\boxed{t = 63 \text{ minutes}}$ amount of water is maximum because of the FIRST DERIVATIVE TEST.

D1

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

pumped – 8 gpm
leaks – $\sqrt{t+1}$ gpm
$t=0$, 30 gallons

$$\int_0^3 \sqrt{t+1}\,dt = \int_0^3 (t+1)^{\frac{1}{2}}dt \qquad u=t+1,\ du=dt$$

$$= \tfrac{2}{3}(t+1)^{\frac{3}{2}}\Big]_0^3$$

$$= \tfrac{2}{3}(4)^{\frac{3}{2}} - \tfrac{2}{3}(1)^{\frac{3}{2}}$$

$$= \tfrac{2}{3}(8) - \tfrac{2}{3}$$

$$= \frac{16}{3} - \frac{2}{3}$$

$$= \frac{14}{3} \text{ gallons}$$

Work for problem 4(b)

$$\frac{8 \text{ gallons}}{1 \cancel{\text{min}}}\left(\frac{3\ \cancel{\text{minutes}}}{}\right) = 24 \text{ gallons} - \frac{14}{3}\text{ gallons} =$$

$$\frac{72}{3} - \frac{14}{3} = \boxed{\frac{58}{3}\text{ gallons}}$$

$24 \times 3 = 72$

$72 - 14 = 58$

Continue problem 4 on page 11.

D_2

Work for problem 4(c)

$$A(t) = 8t - \int_0^t (t+1)^{\frac{1}{2}}\,dt$$

Work for problem 4(d)

$$A'(t) = 8 - (t+1)^{\frac{1}{2}} = 0$$

$$\left(\sqrt{t+1}\right)^2 = (8)^2$$

$$t+1 = 64$$

$$\boxed{t = 63}$$

+ + + + 63 - - -

When t is 63, the graph A(t) reaches a maximum (goes from positive to negative). So, the amount of water is at its maximum in the tank when t = 63.

GO ON TO THE NEXT PAGE.

F1

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$$\int_0^3 \sqrt{t+1}\,dt$$

$$\int_0^3 (t+1)^{1/2}\,dt$$

$$\frac{2}{3}(t+1)^{3/2}\Big]_0^3$$

$$\left(\frac{2}{3}\right)(4)^{3/2}-\left(\frac{2}{3}\right)(1)^{3/2}$$

$$\left(\frac{2}{3}\right)(8)-\frac{2}{3}=\frac{16}{3}-\frac{2}{3}=\boxed{\frac{14}{3}\text{ gallons}}$$

Work for problem 4(b)

$$54-\frac{14}{3}=\frac{162}{3}-\frac{14}{3}=\frac{148}{3}=\boxed{49\tfrac{1}{3}\text{ gallons}}$$

Continue problem 4 on page 11.

F_2

Work for problem 4(c)

$A(t) = \left[(30 + 8t)\right] - \left[\int_0^3 \sqrt{t+1}\, dt\right]$

Work for problem 4(d)

$(t+1)^{1/2}$

$\frac{1}{2}(t+1)^{-1/2} = 0$

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5 5 5 5 5 5 5 5 5 5

A₁

Work for problem 5(a)

(a) $xy^2 - x^3y = 6$

$x(2y)\left(\frac{dy}{dx}\right) + y^2 - 3x^2y - x^3\left(\frac{dy}{dx}\right) = 0$

$\left(\frac{dy}{dx}\right)2yx - \left(\frac{dy}{dx}\right)x^3 = 3x^2y - y^2$

$\frac{dy}{dx}(2yx - x^3) = 3x^2y - y^2$

$\frac{dy}{dx} = \frac{3x^2y - y^2}{2yx - x^3}$

Work for problem 5(b)

$(1)y^2 - (1)y = 6$

$y^2 - y - 6 = 0$

$(y+2)(y-3) = 0$ $(1,-2)$ and $(1,3)$

$y = -2$ or 3

at $(1,-2)$

$\left.\frac{dy}{dx}\right|_{(1,-2)} = \frac{3(1)(-2) - 4}{2(-2)(1) - (1)} = \frac{-6-4}{-4-1} = \frac{-10}{-5} = 2$

$y = 2(x-1) - 2$

$\boxed{y = 2x - 4}$

at $(1,3)$ $\left.\frac{dy}{dx}\right|_{(1,3)} = \frac{3(3) - (3)^2}{2(3) - 1} = \frac{9-9}{6-1} = 0$

$\boxed{y = 3}$

Continue problem 5 on page 13.

A_2

Work for problem 5(c)

(c) $\frac{dy}{dx}$ does not exist, y does exist

& $\frac{dy}{dx}$ does not change sign on either side of pt.

$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$ which D.N.E.

y does exist when $xy^2 - x^3y = 6$

when $2xy - x^3 = 0$

$x(2y - x^2) = 0$

$x = 0$ or $2y - x^2 = 0$

$2y = x^2$

$y = \frac{x^2}{2}$

$(0)y - (0)y = 6$

$0 \neq 6$

$x\left(\frac{x^2}{2}\right)^2 - x^3\left(\frac{x^2}{2}\right) = 6$

$\frac{x^5}{4} - \frac{x^5}{2} = 6$

$\frac{x^5}{4} - \frac{2x^5}{4} = 6$

$-\frac{x^5}{4} = 6$

$x^5 = -24$

$\boxed{x = \sqrt[5]{-24}}$

GO ON TO THE NEXT PAGE.

Work for problem 5(a)

$xy^2 - (x^3y) = 6$

$(x \cdot 2yy') + (y^2) - [(x^3y') + (3x^2y \cdot)] = 0$

$2xyy' + y^2 - x^3y' - 3x^2y = 0$

$2xyy' - x^3y' = 3x^2y - y^2$

$y'(2xy - x^3) = 3x^2y - y^2$

$y' = \dfrac{3x^2y - y^2}{2xy - x^3}$

Work for problem 5(b)

$x = 1$

$xy^2 - x^3y = 6$

$(1)y^2 - (1)^3y = 6$

$y^2 - y = 6$

$y^2 - y - 6 = 0$

$(y-3)(y+2)$

$y = 3, -2$

$m = \dfrac{3x^2y - y^2}{2xy - x^3}$ for $(1, 3)$ $m = 0$

$m = \dfrac{3(1)^2(3) - (3)^2}{2(1)(3) - (1)^3}$

$m = \dfrac{9-9}{6-1} = 0$

so pts are $(1,3)$ and $(1,-2)$

$m = \dfrac{3x^2y - y^2}{2xy - x^3}$ at $(1,-2)$

$m = \dfrac{3(1)^2(-2) - (-2)^2}{2(1)(-2) - (1)^3}$

$= \dfrac{-6-4}{-4-1} = \dfrac{-10}{-5} =$

• so eq^n for $(1,3)$; $y - 3 = 0(x-1)$
$m=0$
$y = 3$

• eq^n for $(1,-2)$; $y + 2 = 2(x-1)$
$m = 2$

Continue problem 5 on page 13.

Work for problem 5(c)

tangent line = vertical when denominator of $\frac{dx}{dy}$ is 0

so $0 = 2xy - x^3$

Work for problem 5(a)

$xy^2 - x^3y = 6$

$x \cdot 2y\frac{dy}{dx} + y^2 \cdot 1 - x^3\frac{dy}{dx} + y(-3x^2) = 0$

$2xy\frac{dy}{dx} - x^3\frac{dy}{dx} = 3x^2y - y^2$

$\frac{dy}{dx}(2xy - x^3) = 3x^2y - y^2$

$\boxed{\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}}$

Work for problem 5(b)

$(1)y^2 - (1)^3y = 6$

$y^2 - y = 6$

$y^2 - y - 6 = 0$

$\frac{1 \pm \sqrt{-1^2 - 4(1)(-6)}}{2(1)}$

$\frac{1 \pm \sqrt{25}}{1}$

$\frac{1+5}{1}$ or $\frac{1-5}{1}$

6 or -4

$\boxed{(1,6) \text{ and } (1,-4)}$

$\frac{3x^2y - y^2}{2xy - x^3} = \frac{dy}{dx}$

$\frac{3(1)^2(6) - 6^2}{2(1)(6) - (1)^3} = \frac{18-36}{12-1} = \frac{-18}{11} = \frac{dy}{dx}$

$y - 6 = -18/11(x-1)$

$\boxed{y = -18/11(x-1) + 6}$

$\frac{3x^2y - y^2}{2xy - x^3} = \frac{dy}{dx}$

$\frac{3(1)^2 \cdot 4 - 4^2}{2(1)(4) - 1^3} = \frac{12-16}{8-1} = \frac{-4}{7}$

$y - 4 = -4/7(x-1)$

$\boxed{y = -4/7(x-1) + 4}$

Continue problem 5 on page 13.

Work for problem 5(c)

$$2xy - x^3 = 0$$

$$2x\left(-\frac{4}{7}(x-1)+4\right) - x^3 = 0$$

$$2x\left(-\frac{4}{7}x + \frac{4}{7} + 4\right)$$

$$-\frac{8}{7}x^2 + \frac{8}{7}x + \frac{8}{1}x = 0$$

$$-\frac{8}{7}x^2 + \frac{64}{7}x = 0$$

$$\frac{8}{7}x\,(x+8) = 0$$

$$\boxed{\begin{matrix} x = 0 \\ x = -8 \end{matrix}}$$

GO ON TO THE NEXT PAGE.

AP Calculus AB
2000 Student Samples

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A1

Work for problem 6(a)

$$\frac{d\cancel{x}y}{dx} = \frac{3x^2}{e^{2y}}$$

$$e^{2y}\,dy = 3x^2\,dx$$

$$\int e^{2y}\,dy = \int 3x^2\,dx$$

$$\frac{1}{2}e^{2y} = x^3 + c \quad \longleftarrow \quad f(0)=\frac{1}{2} \quad \begin{matrix} x=0 \\ y=\frac{1}{2} \end{matrix}$$

$$\frac{1}{2}\cdot e^1 = 0 + c$$

$$c = \frac{e}{2}$$

$$\frac{1}{2}e^{2y} = x^3 + \frac{e}{2}$$

$$e^{2y} = 2x^3 + e$$

$$2y = \ln(2x^3 + e)$$

$$y = \frac{\ln(2x^3+e)}{2}$$

$$\boxed{\therefore\; y = \frac{\ln(2x^3+e)}{2}}$$

$\ln \quad e^{1\,0} \quad e^3$

Continue problem 6 on page 15.

A_2

Work for problem 6(b)

Domain...

$\ln(2x^3+e)$

$$2x^3+e>0$$

$$2x^3>-e$$

$$x^3>-\frac{e}{2}$$

$$\boxed{x>-\sqrt[3]{\frac{e}{2}}}$$

Range...

$$y=\frac{\ln(2x^3+e)}{2}$$

$\therefore$ Domain $D: \{x \mid x>-\sqrt[3]{\frac{e}{2}}\}$

Range $R: \{y \mid y \in \mathbb{R}\}$

END OF EXAMINATION

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C_1

Work for problem 6(a)

$$f(x) = \int \frac{3x^2}{e^{2y}} \frac{dy}{dx}$$

$$= \int e^{2y}\,dy = \int 3x^2\,dx$$

$$= 2e^{2y} = x^3 + C$$

but @ $x = 0,\ y = \frac{1}{2}$

AND $2e^{2(\frac{1}{2})} = 0^3 + C$

$$2e = C$$

$$2e^{2y} = x^3 + 2e$$

$$e^{2y} = \frac{x^3 + 2e}{2}$$

$$2y = \ln\left(\frac{x^3 + 2e}{2}\right)$$

$$\boxed{y = \frac{1}{2}\ln\left(\frac{x^3 + 2e}{2}\right)}$$

Continue problem 6 on page 15.

Work for problem 6(b)

C2

$$\frac{x^3+2e}{2} > 0$$

$$x^3+2e > 0$$

$$x^3 > -2e$$

$$\boxed{x > \sqrt[3]{-2e}}$$

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Work for problem 6(a)

F1

$$e^{2y}\,dy = 3x^2\,dx$$

$$\int e^{2y}\,dy = \int 3x^2\,dx$$

$$\frac{e^{2y}}{2} = x^3$$

$$e^{2y} = 2x^3$$

$$2y = \ln 2x^3$$

$$y = \frac{\ln 2x^3}{2}$$

$$f(x) = \frac{\ln 2x^3}{2}$$

$$f(0) = \frac{\ln 0}{2} = \frac{1}{2}$$

Continue problem 6 on page 15.

Work for problem 6(b)

F2

$\ln x > 0$

$2x^3 > 0$

D: $x > 0$

R: all $\mathbb{R}$

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